Advanced Higher Integrals

Note: nothing is given in the exam – remember everything !

In the following, a and b are real constants and $f(x)$ a real-valued function with no restrictions unless otherwise stated.

Reciprocal Trigonometric Functions

$$\int \sec^2 (ax+b)\ dx = \frac{1}{a} \tan (ax+b) + C$$

$$\left(a \neq 0,\ ax+b \neq \frac{(2m+1)\pi}{2},\ m \in \mathbb{Z} \right)$$

$$\int \sec (ax+b) \tan (ax+b)\ dx = \frac{1}{a} \sec (ax+b) + C$$

$$\left(a \neq 0,\ ax+b \neq \frac{(2m+1)\pi}{2},\ m \in \mathbb{Z} \right)$$

$$\int \operatorname{cosec} (ax+b) \cot (ax+b)\ dx = -\frac{1}{a} \operatorname{cosec} (ax+b) + C$$

$$\left(ax+b \neq m\pi,\ m \in \mathbb{Z} \right)$$

$$\int \operatorname{cosec}^2 (ax+b)\ dx = -\frac{1}{a} \cot (ax+b) + C$$

$$\left(ax+b \neq m\pi,\ m \in \mathbb{Z} \right)$$

Special cases

- $b = 0$ gives

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C$$

$$\int \operatorname{cosec} ax \cot ax \, dx = -\frac{1}{a} \operatorname{cosec} ax + C$$

$$\int \operatorname{cosec}^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

- $b = 0, a = 1$ gives

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + C$$

Inverse Trigonometric Functions

$$\int \frac{f'(x)}{\sqrt{1 - \left(f(x)\right)^2}} \, dx = \sin^{-1} f(x) + C$$

$$\int \frac{-f'(x)}{\sqrt{1 - \left(f(x)\right)^2}}\, dx = \cos^{-1} f(x) + C$$

$$\int \frac{f'(x)}{1 + \left(f(x)\right)^2}\, dx = \tan^{-1} f(x) + C$$

Special cases

- $f(x) = \frac{x}{a} \quad (a \neq 0)$ gives

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}}\, dx = \cos^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

- $f(x) = x$ gives

$$\int \frac{1}{\sqrt{1 - x^2}}\, dx = \sin^{-1} x + C$$

$$\int \frac{-1}{\sqrt{1 - x^2}}\, dx = \cos^{-1} x + C$$

$$\int \frac{1}{1 + x^2}\, dx = \tan^{-1} x + C$$

Exponentials

$$\int f'(x)\, e^{f(x)}\, dx = e^{f(x)} + C$$

Special cases

- $f(x) = ax + b$ gives

$$\int a\, e^{ax+b}\, dx = e^{ax+b} + C$$

- $f(x) = x$ gives

$$\int e^{x}\, dx = e^{x} + C$$

Logarithms

$$\int \frac{f'(x)}{f(x)}\, dx = \ln\left|f(x)\right| + C \qquad (f(x) \neq 0)$$

Special cases

- $f(x) = ax + b$ gives

$$\int \frac{a}{ax + b}\, dx = \ln\left|ax + b\right| + C$$

- $f(x) = x$ gives

$$\int \frac{1}{x}\, dx = \ln\left|x\right| + C$$